

# Divisor Methods for Sequential Portfolio Allocation in Multi-Party Executive Bodies: Evidence from Northern Ireland and Denmark

**Brendan O'Leary** University of Pennsylvania  
**Bernard Grofman** University of California, Irvine  
**Jørgen Elklit** University of Aarhus

*Some proportional representation (PR) rules can also be used to specify the sequence in which each party in a parliament or each member in a multiparty governing coalition is given its choice about (unique) desired resources, e.g., "indivisible goods" such as cabinet ministries or executive positions, thus providing an algorithmic method for determining "fair" allocations. Divisor rule sequencing using the d'Hondt method was recently used to determine the ten cabinet positions in the Northern Ireland Executive Committee created under the 1998 Belfast ("Good Friday") Agreement; and such sequential allocation procedures have been used in some Danish municipal governments, and for determination of committee chairs in the European parliament. Here we examine in some detail the procedures used in Northern Ireland and Denmark, with a focus on special features such as the option in Denmark to form post-election alliances.*

Our focus is on a little known institutional mechanism for "automatically" determining portfolio assignments in multiparty cabinets or other multimember executive bodies where different parties are given different arenas of policy or executive responsibility, i.e., *sequential portfolio allocation based on party seat shares*. The basic idea is that each party's seat-share in the legislature (or relative seat share among the parties in a governing coalition) is used as a measure of its legitimate "claims" to ministerial office. Divisor rules used for proportional representation (PR), such as d'Hondt or Sainte-Laguë, are then used to generate a sequencing procedure that determines exactly which party gets first, second, third, etc. choice of ministries—after agreement is reached on the nature and number of cabinet portfolios. Thus, methods commonly used for the proportional

allocation of parliamentary seats (or committee seats) as a function of party vote share (Hix 1999; Taagepera and Shugart 1989), or to apportion seats on the basis of population to certain predefined units, e.g., states, of the United States (Balinski and Young 1982), can be used to resolve a very different problem of allocation, namely that of ministerial portfolios within a governing coalition, or of designated positions on local government council executives.

*Sequential portfolio allocation* methods deserve attention from scholars interested in the design of institutional mechanisms for the solution of collective choice problems, including those involving ethnic conflict, and multiparty governance. We discuss how a d'Hondt-based sequential portfolio allocation method was used between 1999 and 2002 in Northern Ireland to deal with the allocation of ministries among contentious parties in a region

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Brendan O'Leary is Professor of Political Science, University of Pennsylvania, Stiteler Hall, 208 South 37th Street, Philadelphia PA 19104; 215-573-0645 (boleary@sas.upenn.edu). Bernard Grofman is Professor of Political Science, University of California, Irvine, 3151 Social Science Plaza, Irvine, CA 92697-5100; 949-824-6394 (bgrofman@uci.edu). Jørgen Elklit is Professor of Political Science, University of Aarhus, Bartholins Alle, DK, 8000 Aarhus C, Denmark; 45-89-42-12-59 (elklit@ps.au.dk).

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marked by ethno-national cleavages and profound disagreements about its future political status, and we report on the use of a very similar method that has been used in allocating designated positions on local government council executives in the four largest Danish municipalities for decades.

Sequential divisor allocation methods may be particularly appropriate in situations where there is a climate of distrust and hostility among the parties who must share governing responsibilities, as in a polity deeply divided along religious or ethno-national lines such as Northern Ireland. But their use in the four largest cities in Denmark for the allocation of seats in municipal executive bodies and as a means of allocating committee chairships (and other positions of committee authority) in the European Union parliament (Corbett, Jacobs, and Shackleton 2001, 106–08) suggests that they may also be helpful in more general contexts. Sequential portfolio allocation algorithms offer a number of desirable properties for constitutional design, not least the ability to economize on the bargaining costs otherwise necessary to agree on how to allocate indivisible goods such as ministries (and, directly or indirectly, responsibilities for associated policies) among members of a multiparty governing coalition, or between competing party groupings. Here we compare the variants of the rules for determining ministerial assignments in Northern Ireland and the assignments of city council executive positions in Denmark's second largest city, Aarhus, from both a descriptive and a theoretical perspective.

There are a number of options in constructing sequential portfolio allocation mechanisms. Two of the most important issues are the choice of divisor rule (e.g., d'Hondt, Sainte-Laguë, etc.), and the decision whether to allow for post-election party alliances that will affect the party/coalition seat share sizes used for the sequential allocation procedure. Both choices can substantially affect the outcomes of the sequencing process. Both Northern Ireland and Denmark make use of d'Hondt,<sup>1</sup> but Denmark allows for post-election alliances that can affect the party share calculations, while Northern Ireland does not. The Danish rules, we suggest, provide incentives for larger than minimal winning coalitions, while, in Northern Ireland, a potential cross-community coalition is, in effect, built into the constitutional accord, preventing the exclusion of any major party that wishes inclusion (and is prepared to commit to peaceful and democratic politics) from being denied access to office.

<sup>1</sup>D'Hondt is also used for the allocation of committee chairs to the various party groupings in the European parliament. (Corbett, Jacob, and Shackleton 2001)

## d'Hondt and Sainte-Laguë as Sequential Allocation Methods

Mathematically identical rules for PR are often known under one name in the context of population-based apportionment of seats to fixed geographic or political subunits, i.e., districting, but under another name in the context of electoral formulae to translate party vote share into parliamentary seat allocations. For example, after each decennial census between 1790 and 1830, the method of apportionment used to determine the number of members of the Congress of the United States to be allocated to each state was one proposed by Thomas Jefferson. Exactly this same rule was independently proposed by Viktor d'Hondt in 1878 to determine proportionally party seat shares in a parliament. Similarly, the apportionment method proposed by the celebrated nineteenth-century American legislator, Daniel Webster, and used for congressional apportionment in the United States from 1840 to 1920, is identical to the Sainte-Laguë formula for proportional representation.<sup>2</sup>

Both the d'Hondt/Jefferson and the Sainte-Laguë/Webster rules illustrate a class of PR formulae known as *divisor methods*. If we let the first entry in the  $n$ -tuple be one, each method can be characterized by a unique vector of divisors  $(d_1, d_2, d_3, \dots)$ :  $(1, 2, 3, \dots)$  in the case of d'Hondt/Jefferson;  $(1, 3, 5, \dots)$  in the case of Sainte-Laguë/Webster, such that, if there are  $m$  seats to be allocated, and the  $i$ th unit (or party) has a population (vote) share of  $p_i$ , then the  $m$  highest quotients of the form  $p_i/d_i$  determine how many seats each geographic unit or party receives. For example, if three of the  $m$  highest quotients have  $p_1$  in the numerator, then unit 1/party 1 gets three of the  $m$  seats.

Since some readers may not be familiar with divisor methods, or at least may not be familiar with the way we define them here,<sup>3</sup> we illustrate in some detail how such methods work for purposes of sequencing by considering

<sup>2</sup>The Sainte-Laguë method was first introduced in Latvia in 1922 and used for the interwar period parliamentary elections; it was reintroduced in that country in 1992/93. The only other present uses of pure Sainte-Laguë of which we are aware are New Zealand after the electoral system reform of 1993, and Bosnia-Herzegovina, since 1998. Modified Sainte-Laguë has been used in Denmark, Norway, and Sweden since the 1950s (Grofman and Lijphart 2002).

<sup>3</sup>There are alternative ways to calculate winners under each divisor method. For example, divisor methods can be defined via some ratio of population to representatives (or of votes to seats) which we divide into each unit's population share (each party's vote share). The quotients so obtained are converted to whole numbers by being rounded up or down to a neighboring whole number according to a rule that depends upon on the particular method (Balinski and Young 1982).

**TABLE 1A Illustration of the d'Hondt Divisor Rule**

d'Hondt Divisor	Party A	Party B	Party C
1	<b>.47</b> [1]	.32 [2]	.21 [4]
2	<b>.235</b> [3]	<b>.16</b> [5]	.105
3	<b>.1566</b> [6]	.1067	.07
4	<b>.1175</b> [7]		

Numbers in square brackets indicate the order in which parties win seats; i.e., decided by the declining size of the quotients in question. Under d'Hondt party A wins four seats, party B wins two seats, and party C wins one seat.

**TABLE 1B Illustration of the Sainte-Laguë Divisor Rule**

Ste.-Laguë Divisor	Party A	Party B	Party C
1	.47 [1]	.32 [2]	.21 [3]
3	.1566 [4]	.1067 [5]	.07 [7]
5	.094 [6]	.064	.042
7	.067		

Numbers in square brackets indicate the order in which parties win seats.

the allocation of seven seats among three parties, with vote shares of 47%, 32%, and 21%, respectively, as shown in Table 1a and 1b. Here the d'Hondt and Sainte-Laguë rules give us different allocations. The seven highest entries (highlighted in bold) in Table 1a give us four seats for party A, two seats for party B, and one seat for party C under d'Hondt; in contrast, under Sainte-Laguë, party A now drops to three seats, while the other parties win two each. What is of particular interest is that if we arrange the ratios,  $p_i/d_j$ , from highest to lowest, then the allocation of the  $m$  seats can be thought of as occurring in a particular sequence. If we are allocating seats to political parties, the party associated with the highest quotient from the set  $p_i/d_j$  gets the "first" seat; the party with the second highest quotient gets the "second" seat; and so on.

From this insight, it is easy to think of generalizing this sequencing rule to the allocation of nondivisible units, e.g., ministerial portfolios or local council executives. Now, the party associated with the highest quotient from the set  $p_i/d_j$  can be thought of as being given the "first pick" of ministerial portfolios; the party with the second highest quotient can be thought of as getting the "second pick" among the set of (still open) ministerial portfolios; and so forth. For example, for the data shown in Table 1a, under d'Hondt, party A gets the first, third, sixth, and seventh picks; party B gets the second and fifth, picks; and party C gets the fourth pick. For the Sainte-Laguë

**TABLE 2A Another Illustration of the d'Hondt Divisor Rule**

d'Hondt Divisor	Party A	Party B	Party C
1	.45 [1]	.37 [2]	.18 [5]
2	.225 [3]	.185 [4]	.09
3	.15 [6]	.1233 [7]	.06
4	.11025		

Numbers in square brackets indicate the order in which parties win seats.

**TABLE 2B Another Illustration of the Sainte-Laguë Divisor Rule**

Sainte-Laguë Divisor	Party A	Party B	Party C
1	.45 [1]	.37 [2]	.18 [3]
3	.15 [4]	.1233 [5]	.06
5	.09 [6]	.074 [7]	.036
7	.06042		

Numbers in square brackets indicate the order in which parties win seats.

allocation rule, as shown in Table 1b, in contrast, party A gets the first, fourth, and sixth picks, while party B gets the second and fifth picks, and party C gets the third and seventh picks.

D'Hondt is the most commonly used for European list PR allocations, although neither it nor Sainte-Laguë is the divisor rule currently in use for apportioning the U.S. House of Representatives. Which method is to be used can, as we have demonstrated, have consequences for outcomes. It is well known that d'Hondt/Jefferson tends to be more favorable to larger parties than Sainte-Laguë/Webster. The same pattern also applies for sequencing, i.e., Sainte-Laguë/Webster tends to give smaller parties a better chance at "high" picks than does d'Hondt/Jefferson.<sup>4</sup>

We now provide an example to demonstrate that, even if the d'Hondt and Sainte-Laguë rules allocate the same distribution of seats between parties, they need not have identical sequencing implications. Consider again, for example, an allocation of seven seats among three parties, but now with vote shares of 45%, 37%, and 18%, respectively, as shown in Table 2a and 2b. Here the d'Hondt

<sup>4</sup>d'Hondt/Jefferson and Sainte-Laguë/Webster are, of course, merely two examples of possible divisor methods, the most commonly used of an infinite family of divisor sequences. Other well-known examples include Imperiali (1, 3, 4, 5, 6, ...), modified Sainte-Laguë (1.4, 3, 5, 7, ...), and Danish (1, 4, 7, 10, ...). Indeed, (multiseat) plurality is also a divisor method, with divisor sequence (1, 1, 1, 1, ...).

**TABLE 3 The Initial Shares of Blocs and Parties in the 1998 Assembly**

Bloc	Seats Won (%)
Nationalists	42 (38.8)
SDLP	24 (22.2)
SF (Sinn Féin)	18 (16.6)
Other nationalists	
'Yes' Unionists	30 (27.7)
UUP	28 (25.9)
PUP	2 (1.8)
UDP	—
Other yes unionists	—
'No' Unionists	28 (25.9)
DUP	20 (18.5)
UKUP	5 (4.6)
Other no unionists	3 (2.8)
Others	8 (7.4)
APNI (Alliance)	6 (5.5)
WC	2 (1.9)
TOTALS	108 (100)

and Sainte-Laguë rules give us identical allocations. But, whereas under d'Hondt, party A gets the first, third, and sixth picks, party B gets the second, fourth, and seventh picks, and party C gets the fifth pick, by contrast under the Sainte-Laguë rule party A gets the first, fourth (rather than third), and sixth picks, while party B gets the second, fifth (rather than fourth), and seventh picks, and party C gets the third (rather than fifth) pick. The switch to Sainte-Laguë favors the smallest party.

## The Use of Divisor Rules in Northern Ireland and Denmark

### Results of the 1998 Election in N. Ireland

According to the procedures agreed upon in the 1998 Accord, the Assembly members are to be elected by the single transferable vote in 18 six-member districts and, upon election, are required to designate themselves into one of three groups: *nationalist* (i.e., against continuing union with Great Britain), *unionist*, and *other*. The election results of June 1998 are shown in Table 3.<sup>5</sup> After the elections, 42 Members of the Northern Ireland Assembly

<sup>5</sup>Election results for the groupings quite closely mirrored first pick preferences on the STV ballots. (Note: One member of the UUP subsequently refused party discipline and, in effect, became a "No Unionist.")

**TABLE 4 The d'Hondt Rule and the Distribution of Ministries in Northern Ireland, November 1999**

d'Hondt Divisor	UUP		SDLP		DUP		SF	
	S	M	S	M	S	M	S	M
1	27	[1]	24	[2]	20	[3]	18	[4]
2	13.5	[5]	12	[6]	10	[7]	9	[9]
3	9	[8]	8	[10]	6.7		6	
4	7		6		5		4.5	
TOTAL PICKS		3		3		2		2

Key: S = Seats, M = Ministries, numbers in square brackets indicate sequencing of picks.

Note: The UUP had one less MLA than shown in Table 3, because one of its members had the party whip withdrawn. In effect, this member shifted from the unionist "Yes" camp to the unionist "No" camp.

(MLAs) were *nationalist*, 58 were *unionist*, and eight identified themselves as *other* (those who did not wish to accept either a "nationalist" or "unionist" label). The two major groups of nationalists in this election were the SDLP and Sinn Féin (militant republicans). The unionists can be divided into two major groupings, the *Yes Unionists*, who supported the 1998 Agreement: the two major parties in this bloc were the Ulster Unionist Party (UUP) and the Progressive Unionist Party (PUP); and the *No Unionists*, who opposed the 1998 Agreement: the two major parties in this bloc were the Democratic Unionist Party (DUP) and the United Kingdom Unionist Party (UKUP)—which has since split. In addition, there were independent unionists. The *others* consisted of the Alliance Party and the Women's Coalition.

The parties reached an agreement to establish ten ministries and specified the title and areas of responsibility of each. We show in Table 4 the application of the d'Hondt rule to the distribution of picks for these 10 ministries, but we have not bothered to show results for the smaller parties which did not receive any ministries.

Once the number of ministries was determined each party's number of ministerial portfolios was known in advance,<sup>6</sup> but it was not yet known whether all parties would take up their entitlements. In fact, in the first attempt at implementing the allocation procedure, in July

<sup>6</sup>As can be seen from the table, the Alliance, with five seats, was not large enough to earn a ministerial pick, since the tenth allocation went to the SDLP, with a quotient of 8. In the case of ties, the party with the higher share of first-preference votes in the Assembly election is given precedence in portfolio choice. As seen in Table 4, such a tie actually occurred (see the two quotients of size 9). UUP had more first place votes in the STV balloting than SF and received the eighth pick.

**TABLE 5A Party Choices of Ministerial Portfolios, July 1999 (Failed Run)**

Ministerial Portfolio in Order Chosen	Party	Nominee	Notes
1. Finance and Personnel	SDLP	Mark Durkan	UUP absent
2. Enterprise, Trade, and Investment	SF	Bairbre de Brún	DUP refused
3. Regional Development	SDLP	Sean Farren	
4. Agriculture and Rural Development	SF	Martin McGuinness	
5. Higher and Further Education, Training, and Employment	SDLP	Brid Rogers	
6. Education	SF	Pat Doherty	
7. Health, Social Services, and Public Safety	SDLP	Eddie McGrady, then Joe Hendron	McGrady refused party leader's nomination
8. Social Development	SDLP	Denis Haughey	
9. Culture, Arts, and Leisure	SF	Mary Nelis	
10. Environment	SDLP	Alban Maginnis	

1999, a boycott by the unionist parties and the Alliance (triggered by a dispute about the pace of demilitarization) led to the outcome that six ministerial portfolios went to the SDLP and four to Sinn Féin. Anticipating such a politically infeasible outcome (i.e., all seats in the executive held by one grouping), the U.K. Secretary of State for Northern Ireland had promulgated an emergency Standing Order that voided the portfolio allocation procedure outcomes if they did not result in an executive body with “at least three designated Nationalists and three designated Unionists.”<sup>7</sup> A compromise was reached in November 1999, which permitted executive formation to take place involving all parties with representation in the Northern Ireland Assembly, thus generating the sequencing allocation shown in Table 4.

For purposes of comparison with the “real” portfolio allocation, we show the results of the “failed” allocation procedure of July 1999 in Table 5a. In Table 5b we show how each of the four parties with ministerial representation used their picks in November 1999.

### Empirical Questions about the Nature of Party Utility Functions in Northern Ireland

This case prompts at least three theoretically important empirical questions:

- (1) Are party preferences for portfolios identical to one another, i.e., can we order ministries by some single metric, e.g., prestige, in such a way that all parties have the same ranking ordering of preferences over ministries? If so, then once the election results are in, and thus the order of picks is known, it will be obvious which parties will have which ministries.
- (2) Even if party preferences for portfolios are not identical to one another, it remains a question as to whether each party's preferences for ministries are *separable*, i.e., is a given party's preference ranking for a given ministry independent of what ministries have already been (or are expected to be) allocated to other parties?<sup>8</sup> When preferences are separable, we may act as if parties simply have a rank ordering of posts—even if not all parties have the same rank ordering.
- (3) If party preferences for portfolios are not identical to one another, but are separable, another important question is whether or not all parties will vote *sincerely* in making ministerial choices, where voting sincerely means to pick the portfolio among those remaining open that is highest on one's preference list. If parties believe that they know the preferences of other parties, and imagine that those preferences are not identical to

<sup>7</sup><http://www.ni-assembly.gov.uk/record/990715.htm>, 15 July 1999.

<sup>8</sup>In the portfolio allocation context, an agent's preferences over two potential objects of choice 1 and 2, are separable if, say, 1 is preferred to 2 regardless of what happens to possible objects of choice 3, 4, 5, and so on. (While this definition is adopted to the particular context of our study, it is effectively the same as the standard definition of “separability”: see e.g., Kadane 1972).

TABLE 5B Party Choices of Ministerial Portfolios, November 1999 (Valid Run)

Ministerial Portfolio in Order Chosen	Party	Nominee	Notes
1. Enterprise, Trade, and Investment	UUP	Reg Empey	
2. Finance and Personnel	SDLP	Mark Durkan	
3. Regional Development	DUP	Peter Robinson	
4. Education	SF	Martin McGuinness	
5. Environment	UUP	Sam Foster	
6. Higher and Further Education, Training, and Employment	SDLP	Sean Farren	
7. Social Development	DUP	Nigel Dodds	Time-out called before choice
8. Culture, Arts, and Leisure	UUP	Michael McGimpsey	
9. Health, Social Services, and Public Safety	SF	Bairbre de Brún	Time-out called before choice
10. Agriculture and Rural Development	SDLP	Brid Rodgers	

their own, there may be incentives for insincere voting. Imagine, for example, that party A has been given round one and round three picks, but is confident that the party with the second round pick has no interest in the ministry which is first on party A's preference list, but instead, ranks in first place the ministry which party A ranks second. Now, party A, to be clever, might pick its second choice first, in anticipation of being able to get its first choice when it uses its third-round pick.

We will provide some (partly impressionistic) evidence on each of these questions derived from the 1999 Northern Ireland sequential allocation process. We proceed by attempting to reconstruct party preferences: using a mixture of the Assembly Record,<sup>9</sup> comparisons of outcomes in the November 1999 process to the those of the failed July 1999 picks (where only two parties were represented in the executive), and informed surmise based in part on confidential interviews carried out by one of the present authors in 2000 with party representatives in Belfast and London.

Let us first consider concordance among party rankings of ministerial importance. On the one hand, there is solid evidence that some ministries are especially highly

(albeit not identically) ranked by all parties and others relatively lowly ranked. For example, Finance and Personnel and Enterprise, and Trade and Investment, are widely regarded as the most important economic ministries, and they were chosen as first or second picks in both July and November 1999, even though different parties were doing the choosing in the two periods. Also, the same third choice, Regional Development, was made in both periods. Similarly, Social Development, Culture, Arts and Leisure, and Health, and Social Services and Public Safety, are each ranked between seventh and the ninth picks in both July and November. On the other hand, there is also clear evidence that there were party-specific idiosyncrasies in the rankings given to the various posts. For example, the Pearson  $r$  for the correlations between the rankings of ministries in July and November is only .58.<sup>10</sup> Thus we observe a mix of ministerial prestige/power considerations and considerations that are particular to given parties—differences that may be triggered by which constituencies/interests are strongly represented in party councils,<sup>11</sup> or by the ambitions/interests of influential

<sup>9</sup>For the Assembly Record see <http://www.ni-assembly.gov.uk/record/991129.htm>, 29 Nov. 1999.

<sup>10</sup>Here the Spearman rank order coefficient and the Pearson correlation coefficient will, of course, be identical. This value is not significant at even the .05 level, but that failure of statistical significance is essentially due to the small  $N$  ( $N = 10$ ).

<sup>11</sup>Despite extensive theoretical work on cabinet coalition formation, until recently, little was known empirically about the actual

members within the party,<sup>12</sup> or by the existence of party preferences that are conditional on the observed or expected choices of other parties.

Now we turn to the question of separability. If preferences are nonseparable, then we would expect that parties about to make a pick would sometimes ask for “time-outs” to confer with their members, and this would be most likely when they were confronted with scenarios they had not previously anticipated. This occurred twice during the November 1999 portfolio assignments (see Table 5b).<sup>13</sup> Other evidence of nonseparability comes from examination of the choice of education ministries. We believe that one plausible explanation of Sinn Féin’s apparent preference reversal between July 1999 and November 1999 in its ranking between Agriculture ahead of Education (having placed the former over the latter, when both were still feasible, in July; but reversing that preference in November) strongly suggests an interdependence of a party’s preferences, and its beliefs about what will happen to a given ministry if it goes into other hands. In July 1999, Education was going to be either in the hands of the SDLP or of Sinn Féin, and it may have mattered little to the latter which of these outcomes occurred; in contrast, in November 1999, if Education was not held by a Nationalist (Catholic) party, then it would be held by a Unionist

bargaining processes that generate coalitional agreements for multiparty cabinets in democratic societies. Before the work of Budge and Keman (1990), there was little comparative analysis of ministerial preferences across political party types, or the match between parties’ ministerial preferences and coalition outcomes. Budge and Keman found solid evidence that parties predictably demand control over particular ministries directly related to their functional constituencies or ideological orientations. For example, religious parties in Israel seek to control the ministries with responsibility for family and marriage, and Scandinavian agrarian parties have regularly won control of the Agriculture and related ministries. Relevant discussion may be found in the country-specific chapters in Doring (1996) and Muller and Strom (2000).

<sup>12</sup>For example, Environment was neither a highly prestigious ministry nor a particularly high priority for the UUP, and thus UUP’s decision to use its second (round five) pick to place Sam Foster as Minister for the Environment is likely to be due to the local interests of that particular party member. Environment contains local government within its remit, and Foster was a local councillor who may have wanted this portfolio as the one closest to his experience. The choice of this ministry may also partly have been motivated by a desire to block unwelcome changes that might be proposed for local government by nationalists, but Foster was almost certainly being rewarded by his party leader for delivering crucial support in a recent leadership battle.

<sup>13</sup>Recall that if preferences are separable then nothing any other party did could influence a party’s own ranking of the ministerial portfolios. Thus, there would never be any need for time-outs.

(Protestant) party, and this may well have been seen by Sinn Féin as important.<sup>14</sup>

Further evidence of interdependencies (nonseparability) comes from post-allocation evaluations of the consequences of the allocation of the two education portfolios in November 1999. As noted above, the first education ministry went to Sinn Féin on round four. Whatever the rationale for the UUP’s choice of Environment on round five, it left the second education portfolio (Higher education plus employment training) still open on round six, where it was chosen by the SDLP.<sup>15</sup> Subsequently, the DUP claimed that the UUP’s decisions—both to make the Agreement, and its choices of portfolios—had left nationalists in full control of education throughout Northern Ireland.<sup>16</sup> Clearly, at least upon hindsight, the value of the second education ministry to the unionist bloc changed after the first education ministry had come under the control of the nationalists.

A related issue of interdependency arises with the Social Development portfolio. After the time-out it had called the DUP used its seventh round pick to choose Social Development. This ministry combines housing, urban, and welfare services. The DUP’s choice left the UUP with a major headache. If it took Health, Social Services, and Public Safety or Agriculture, the two remaining big spending ministries, then it would leave a nationalist party—either SDLP or Sinn Féin—in charge of the ministry of Culture, Arts, and Leisure. This ministry could develop significant agenda-setting control over items such as parades, public symbolism, and language policy. We

<sup>14</sup>There is also the possibility that Sinn Féin was deliberately misrepresenting its preferences vis-à-vis Agriculture and Education in July 1999, in anticipation of fooling other parties about its true preferences when (and if) a “valid” running of the sequential allocation procedure were to take place. From the Assembly transcript we know that Sinn Féin’s choice of Education as its first pick in November 1999 seems to have taken unionists greatly by surprise and to their dismay. Indeed, Sinn Féin may have chosen Education as an affront to the UUP.

<sup>15</sup>This choice might have been predicted from the July 1999 choices of the SDLP. The two ministries, which we know the party then ranked higher when the Higher Education Ministry was available, were no longer free to be chosen in November. Nonetheless, when this choice was made by the SDLP the DUP asked for a time out before its seventh round pick.

<sup>16</sup>Of course, the seriousness of the control of nationalists of the two ministries of education was exaggerated by the DUP, given the checks and balances in the Assembly’s committee systems and decision-making rules, but it was still powerful rhetoric. Moreover, the attack on the UUP was strongly misleading because the DUP could have picked either education portfolio for itself at stage three of the allocation, but did not do so. Indeed, the DUP may have deliberately foregone choice of an education ministry so as to embarrass the UUP.



believe the UUP chose to sacrifice access to one of two remaining big spending ministries by picking Culture, Arts, and Leisure to avoid giving the nationalists the possibility of power over such highly visible policy decisions.<sup>17</sup>

Finally, we consider the evidence for strategic voting in the ministerial selection process in Northern Ireland in 1999. The first and most obvious point is that the retrospective unhappiness of the DUP with the resulting control of the two education ministries by the nationalists suggests that parties were not fully able to “think ahead,” and that they allowed picks to be made by other parties that they subsequently rued, but that they clearly could have prevented. On the other hand, we believe that the UUP’s choice of Culture, Arts, and Leisure over Health, Social Services, and Public Safety or Agriculture, in order to prevent that ministry from coming under nationalist control, comes under the heading of strategic foresight.

## Denmark

### Use of Sequential Allocation Rules by Local Governments in Denmark

Sequential portfolio allocation using d’Hondt divisors has been practiced in Copenhagen since 1938 for allocation of positions in the executive body of the city. The same sequencing system was introduced in 1950 in Aarhus; later the system was also implemented in Odense and Aalborg. In each of these four major cities, the allocation of seats to the executive body of the city council is based on the number of seats held by the political parties.<sup>18</sup> In all Danish cities, the mayor is selected by vote of the members of the council by a simple majority.<sup>19</sup> Five-to-seven po-

sitions on the executive body are then allocated<sup>20</sup> by sequential methods, using the d’Hondt mechanism.<sup>21</sup> Each position is associated with specific policy responsibilities (e.g., social welfare and health, schools and parks, building and planning, public transportation, etc.). Although the mayor is chosen by a simple majority vote, for purposes of deciding the order of choices on the executive body positions, the coalition which elected the mayor is considered to have used their “first pick” to do so. Thus, in a city like Aarhus where, since 1985, there have been five executive body seats, there are actually six different quotients that are determinative of the allocation of posts under the d’Hondt sequential process.

### The Role of *Apparentement*

In all four Danish cities using sequential allocation rules, parties may, prior to the allocation of executive positions, choose to form voting coalitions, a form of *apparentement*; and it is the combined total seat share commanded by each single-party or party coalition bloc that is used to determine the allocational sequencing (Berg and Petersen 2001). In these cities, post-election coalitions (*apparentement*) can and does change the outcomes of the allocation process. Because it is rare for a single party to have the majority of the seats needed to elect the mayor, the period before the first council meeting is one of intensive, and often rather tense, private negotiations among the parties (Pedersen 1997). Very often, the parties to the major(ity) voting coalition formalize their agreement by signing a written covenant, which indicates how posts and spoils and side-payments are to be allocated among coalition partners.

The potential for post-election coalitions to form can significantly affect the incentive structure for parties looking to be part of a governing coalition. From standard theoretical approaches, we might expect ideologically connected coalitions to form (Axelrod 1970), or we might

<sup>17</sup>The UUP did not call a time-out to consider its eighth-round choice, but it did not need to do so since it would have had time to consider its options when the DUP called a time-out after round six.

<sup>18</sup>The entities seeking office under the d’Hondt list PR system used for Danish local elections may be either national political parties presenting local lists or local groups of concerned citizens (Elklit 1997, 2002b).

<sup>19</sup>In addition to the defined executive body positions and the mayor, the city councils in Aalborg, Aarhus, Copenhagen, and Odense elect two deputy mayors. However, these are primarily ceremonial posts, elected by a separate (sequential allocation) mechanism, and we will disregard such posts in the discussion that follows. However, we would note that these posts may serve as “side-payments” in the bargaining games that determine which post-election coalitions will form and how they will allocate the ministerial picks their combined weight entitles them to among the members of the coalition. (In most other smaller cities in Denmark there is only one deputy mayor, who is elected by majority.) In addition to the

allocations of deputy mayorships and other types of posts not on the executive body, forms of side-payment might include agreements about particular policy issues or about procedural aspects of the council process during the next term. An example of such an agreement, from the City of Aarhus, is available (in Danish) from the third-named author of this article upon request.

<sup>20</sup>In Aarhus, in 1978 and earlier, there were only four positions on the executive body in addition to the mayor.

<sup>21</sup>In the case of ties (i.e., identical quotients), a lot has to be drawn. This is a rare but not unknown phenomenon (e.g., such a tie occurred in Aarhus in 1978). Because of the uncertainty about outcomes caused by identical quotients, the bargaining processes among the parties usually try to avoid scenarios that require a tie-breaking mechanism.



**TABLE 6 Results of the 1985 Aarhus Municipal Election**

Parties	Seats
A. Left Socialists	1
B. Socialist People's Party	7
C. Social Democrats	12
D. Social Liberals	1
E. Greens	1
F. Conservatives	7
G. Liberals	2

expect coalitions at the local level that mimic those at the national level. Looking, however, at the specific sequential allocation algorithm in use in these Danish cities, we might instead (or, in addition) expect parties to join a coalition whose combined weight will allow them to achieve “higher picks” than what they might otherwise have gotten. Since the weights of all parties are known, and the number of posts is fixed (although there are side-payment possibilities), the bargaining game is such that, by and large, actors can calculate in a straightforward way what they might have to gain by being a member of some particular coalition. (Of course, there still must be bargaining within the coalition to determine which parties in it are given which of the picks won by the coalition as a whole.)

To make this abstract discussion more concrete, we will illustrate the process described above by examining the election outcomes and the coalitional results of post-election bargaining following the 1985 municipal election in Aarhus.<sup>22</sup>

### Coalitional Bargaining in Aarhus in 1985

There were seven parties winning seats in the 31-seat city council in Aarhus in 1985, and three others that contested the election who received at least 1% of the vote, even though they won no seats. We may roughly array the seven-seat winning parties on a line from left to right so as to check whether or not coalitions are connected. Table 6 shows the left-most party at the top and the right-most

<sup>22</sup>In Aarhus, one also finds the interesting exception to the rules about sequential allocation that an outgoing member of the executive who is reelected to the executive is allowed to keep his (her) former post if he/she so wishes, so that the party from which that person comes is committed to using one of its (nonmayoral) picks to obtain that post, and no other party may claim the post.

**TABLE 7 Seats Needed to Guarantee  $i$  Picks for  $s = 31$  and  $m = 6$** 

$i$	1	2	3	4	5	6
Seats needed to guarantee $i$ picks	5	9	14	18	23	27

party at the bottom, with seat share for each party given in the next column.<sup>23</sup>

The coalition that initially formed in 1985 was the connected coalition of {C, D, E}. This coalition had only 14 seats—a number insufficient to elect a mayor. Eventually, after much hard bargaining over which party would get the second pick, this coalition was joined by the Conservatives, Party F, creating a connected coalition of size 21. This coalition was not, however, minimal winning, since only 16 votes were needed to elect the mayor, and the votes of parties D and E were not needed to achieve this end.

To begin to get a sense of coalitional incentives in Aarhus in 1985 we can create a table showing the picks that would be received by each party if the remaining parties formed a bloc. Here we make use of the formula that a party of size  $k$  in a legislature of size  $s$  has strength enough to guarantee itself of at least  $i$  of  $m$  seats in an executive body, against even a unified opposition, if

$$k/i > (s - k)/(m - i + 1), \quad (1)$$

which simplifies to

$$k > (is)/(m + 1). \quad (2)$$

In Aarhus in 1985, we have  $s = 31$  and  $m = 6$ . Thus, as  $i$  runs from 1 to 6, we can calculate the minimum seats a party must hold to be able to achieve  $i$  picks against a unified opposition. The “magic numbers” are given in Table 7 below.

Thus, for example, by combining the information in the Table 7 with our previously reported data on seat results for each of the parties in 1985, we can determine that only three of the seven-seat winning parties had enough

<sup>23</sup>The left-right ordering of these parties is the generally agreed order among specialists on Denmark, based on ideological and policy self-placement by voters and party members, over-time voting behaviour in parliament, and expert assessments (see Andersen 1986, 170; Bille and Elklit 2003; Laver and Schofield 1991, 25; Nannestad 1989; Bille and Elklit 2003). Parties F and G (Conservatives and Liberals) are often found to be close to one another, and their relative position may vary from one study to another and from one period to another. For the period under scrutiny and for this particular municipality, the order is as indicated in Table 6.

seats to guarantee itself at least one pick if the remaining parties were to unite against it, and only one of these three had enough seats to guarantee themselves at least two picks if the remaining parties were to unite against them.<sup>24</sup> We can extend such calculations, primarily interesting from a theoretical perspective, by considering all possible coalitions of size  $k$  (again assuming a unified opposition), as  $k$  runs from 1 through 6. In general, we would expect such calculations to provide strong incentives for coalitions to form. We can illustrate this point for Aarhus in 1985.

C and D had been in pre-election coalition, and, although this agreement is not legally binding in the post-election phase, they chose to remain together in the post-election phase. C alone would get picks 2 and 4, while D alone would be entitled to no picks; but the combination of the two provided no further gains (see Table 7). This two-party coalition then expanded to add E to become a three-party coalition totaling 14 seats. The new enlarged coalition was now entitled to picks 2, 4, and 6, an improvement over what the individual parties could have guaranteed themselves, but not enough of a gain to make *all three* parties in the coalition better off, especially since the coalition did not command a majority sufficient to elect a mayor. At this point, bargaining was intense, and eventually this three-party proto-coalition added F, to total 21 seats, and thus become large enough to elect a mayor. Now, the coalition had four picks: picks 1, 2, 4, and 5. While this was not a minimal winning coalition, it was a minimal connected winning coalition.<sup>25</sup> The process stopped here.<sup>26</sup>

The agreement of C, D, E, and F to join together meant that the remaining parties did not form a connected coalition. Yet—we would argue, because of the incentives generated by the sequential allocation rules—these three parties (A, B, and G) did choose to enter the lists as a combined entity. Together (with 10 seats) they were entitled to two positions: picks 3 and 6; had they entered the allocation process as separate parties, two of

<sup>24</sup>Of course, this is obviously a theoretical argument, as the position of the parties on the left-right political spectrum meant that not all combinations were politically viable.

<sup>25</sup>The other minimal connected winning coalition was AB.

<sup>26</sup>If we examine the calculations of the 21-member bloc, we see that, while the addition of party G (with two seats) would have allowed the new coalition to gain an additional pick, there was no gain for the existing coalition members since the new pick would be pick 6. On the other hand, while adding B would have added picks, the smaller parties in the {C, D, E, F} coalition might have reason to not want to have a new entrant larger than they who might make a claim on resources, and B was large enough to gain a pick even if alone.

the parties would get no picks, while party B would only have gotten pick 4.

In each of the three other years we have looked at Aarhus local elections, we also find a larger than minimal winning coalition forming. Indeed, in 1989 the coalition that elected the mayor and took all the seats in the executive contained parties with 29 of the 31 seats in the council. Similar patterns of supraminimal coalitions on local governmental bodies are also found in other of Denmark municipalities, not only in the largest cities (see Pedersen 2000). It sometimes even happens that all (or virtually all) parties and lists join the dominant voting coalition under formation.<sup>27</sup>

## **Comparisons of the Formal Aspects of the Northern Ireland and the Aarhus, Denmark Sequential Procedures Choice of Divisor Rule**

d'Hondt is used in both our cases. In Northern Ireland, confidential interviews with political figures conducted by one of the present authors after April 1998 indicate that the two largest parties settled on d'Hondt as the method for sequencing portfolio picks for three reasons: (1) their familiarity with d'Hondt in committee-share allocation in the European Parliament, (2) enlightened self-interest,<sup>28</sup> and (3) their desire to include other significant parties, or at least not exclude them.<sup>29</sup> In Aarhus, d'Hondt was

<sup>27</sup>The divergence between Denmark's national and local coalitional structures we will call "Pedersen's puzzle," after the Danish political scientist, Mogens N. Pedersen (Pedersen 2000), who apparently was the first to write about the disjunction between the larger-than-minimal winning coalitions forming in Danish local politics and the national parliamentary pattern in Denmark favoring minority governments (Strom 1986). We believe that this disjunction can be accounted for in large part by the special nature of the rules structuring coalitional incentives in Danish local elections. (For a quite different explanation of larger-than-minimal coalitions at the local level in Spain, see Colomer and Martinez 1995.) For reasons of space, a fuller discussion of the theoretical implications of the post-election *apparentement* for Danish coalitions arrangements at the municipal level (and of Pedersen's puzzle) must be left to subsequent work.

<sup>28</sup>The UUP and the SDLP were, respectively, the largest unionist and nationalist parties in Northern Ireland, and we know that each party's negotiators were familiar with the idea that d'Hondt was better than Sainte-Laguë for larger parties.

<sup>29</sup>In 1993, a U.K. Institute of Public Policy Research (IPPR) team, acting as advisors to the U.K. Labour Party's opposition frontbench, outlined for discussion a general model of U.K. and Irish shared authority for Northern Ireland that explicitly suggested the possibility of rank ordering the right to nominate to ministerial portfolios

adopted before the national switch from d'Hondt to modified Sainte-Laguë as the method used for the first tier (regional) PR allocation at the national level (see Elklit 2000a, 18–19).

### Determining the Number of Ministerial Posts to be Allocated

In the executive bodies in Northern Ireland (in 1998) and Aarhus (prior to 1985), considerations of anticipated advantage to parties with decision-making power influenced the decisions about how many (and which) posts were to be allocated under the sequential allocation procedures.

**Northern Ireland.** In 1998, the UUP and SDLP initially agreed that, in addition to a dual premiership, the rest of the cabinet would consist of “at least six and not more than ten” other ministers. In protracted post-Agreement negotiations, principally between the two largest parties, the UUP and the SDLP, it was agreed that there should be ten such Ministers—but this agreement was reached only *after* the election results were known. Thus, agreeing to the number of portfolios did not involve a Rawlsian “original position” in which the parties were in the dark about their prospective futures—because the parties could readily calculate the number of ministries to which they would be entitled after the total number was agreed. It is our view that calculations of relative advantage for the two largest parties under the given sequential allocation rules determined the decision to have 10 ministries.

With six ministries, and all parties availing of their entitlements, the SDLP and UUP would obtain two each, and the DUP and Sinn Féin would obtain one each, an equally balanced cabinet of three unionists and three nationalists. With seven ministries, and all parties availing of their entitlements, there would be an overall unionist majority, and the additional ministry would have gone to the DUP—unacceptable to the SDLP. With eight ministries, and all parties availing of their entitlements, there would have an even larger unionist majority—even less acceptable to the SDLP. With nine ministries, and all par-

using a divisor rule—proposing that this sequencing be done by way of the Sainte-Laguë rule on the basis of vote shares won by party nominees to the executive presidency (O’Leary et al. 1993: chapter 4, esp. 31, and 139–44; see also McGarry and O’Leary 1995, 373–75). Arguably this is the first time such a sequential algorithmic procedure was suggested for specifying Northern Ireland ministerial assignments. These authors had advocated Sainte-Laguë with the aim of ensuring inclusive institutions in which smaller parties (principally hard-line Irish Republican and British loyalists, on the one hand, and the interethnic center, on the other) would believe that they had a fair share. (These authors and others borrowed from ideas in Rose 1976.)

ties availing of their entitlements, the unionist majority would be reduced, but the additional nationalist minister would be from Sinn Féin—not especially agreeable to the SDLP. With 10 ministries, and all parties availing of their entitlements, there would again be a balanced cabinet, of five unionists (three from UUP and two from DUP) and five nationalists (three from SDLP and two from Sinn Féin). Given the SDLP’s veto power over the size of the cabinet—which had to be agreed by cross-community consent—this meant that choice of the size of the cabinet was effectively reduced to one of either six or ten.

Why ten rather than six? Given that the UUP and the SDLP would have had more power within a six-member executive (one-third of the ministries each) as opposed to a 10-member executive (three-tenths of the ministries each), we may surmise that a wish either to bind other parties into the new order or to maximize the number of desirable offices available to their own members motivated the agreement to have the larger cabinet.

**Aarhus, Denmark.** In municipal governments in Denmark, the number and content of positions are usually fixed well in advance of an election and customarily stable for a very long time. A change in the size of the executive body in Aarhus occurred in 1981. In 1978, there were only four members of the executive body in addition to the mayor. A fifth was added in 1981. As with Northern Ireland, we argue that the reasons that politicians in Aarhus opted to increase the size of the city’s executive body can be linked to calculations of strategic advantage under the sequential allocation rules in use. In 1978 the 20-member controlling coalition only qualified for three of the five executive positions (including the mayor); yet they were able to calculate that, had there been six seats, with control of 20 seats they would have won four of the six. They implemented this change for the next election.<sup>30</sup>

### Basis of Eligibility for Ministerial Positions

In both of our research sites, all parties are entitled to seek representation in the executive body if they (or if, in Denmark, the post-election blocs they are part of) are large enough to obtain representation. This is quite different from the usual processes for choosing members of cabinets, where membership is normally limited to parties that are in the governing coalition.<sup>31</sup>

<sup>30</sup>However, this coalition split into two in the next election and so was unable to collectively benefit from its strategic insight.

<sup>31</sup>Also, in neither country do we have a notion of “collective” ministerial accountability.

### Possibility of Forming Pre-Election Coalitions That Will Affect Ministerial Picks

Danish local elections allow for pre-election alliances for purposes of determining seat allocations; elections to the Northern Ireland Assembly do not allow for such vote pooling arrangements.

### Possibility of Forming Post-Election Coalitions That Will Affect Ministerial Picks

In Northern Ireland, there were institutional rules prohibiting parties from forming post-election pacts in order to try to win a higher share of, or better sequencing of, ministerial portfolios. In contrast, in Danish local government councils, parties can join forces (by *apparentement*) to increase their probability of taking the executive portfolios (or, in some councils, committee chairmanships) they fancy.

### Tie-breaking Rules

In Northern Ireland ties are broken in favor of the larger party (based on first preference votes under STV); in Aarhus, they are resolved by lot.

### Speed of Reaching Coalitional Agreements under a Sequential Allocation Algorithm

**Northern Ireland.** In Northern Ireland, once an election is over, if the number and specification of posts is given (and assuming all parties decide to stay “within the system”), the party distribution in the executive is determined; although the specific posts each party will receive remains to be determined by how each party exercises the picks the d’Hondt portfolio algorithm has assigned them. Thus, in principle, the parties should be able to conduct allocations under this algorithm quickly, and this was indeed the case in Northern Ireland once agreement to implement the Agreement’s executive formation was reached in November 1999.

**Aarhus, Denmark.** The speed of reaching coalitional agreements to set up an executive is going to be slowed in situations, such as that in Aarhus, where the nature of the blocs to whom picks are being allocated becomes itself a matter of negotiation among the parties even after the number of seats in the executive body is known. This is true despite the fact that the implications of any coalitional arrangement for the distribution of picks can

readily be calculated, because there is still a lot of room for exercising negotiation skills. While the formal process itself is quick (and is sometimes reduced to reading out the resulting names when the outcomes are clear to all parties), the coalition formation can be time-consuming to achieve.

### Level of Satisfaction with the Ministerial Portfolio Assignment Process

In Northern Ireland in 1999, even though though there was no mechanism for ratifying the portfolio assignments in toto, there was a general feeling that the relative importance of the various parties was being reflected, and the divisor process itself worked smoothly.<sup>32</sup> Similarly, for Aarhus, our conclusion is that the sequential allocation algorithm system in use there has generally worked very well as an instrument that has added both transparency and order to the negotiations over posts in the executive body. The consequences of forming this or that voting coalitions have been clear to all parties, the bargaining strength of each player has been calculable, and the sequencing system has allowed parties to get a clear picture of the main spoils stemming from different kinds of voting and coalition behavior.

### Potential for Initial Portfolio Assignments to Be Overturned by Subsequent (Mutually Beneficial) Vote Trades

*Trade resistance* deals with the question of whether parties have any reason to make trades with one another after all ministerial picks have been exercised. When allocations are not trade resistant than the process is not an efficient one and its fundamental fairness is also cast into doubt. However, parties could not, in Northern Ireland, make trades of ministerial portfolios with one another after the allocation was complete, and a similar requirement to apply and abide by d’Hondt obtains in executive assignments in Danish local governments.

Because we lack sufficiently detailed knowledge about party preferences, for neither country can we directly examine trade resistance. Nonetheless there is a great deal to be said about trade resistance from a theoretical perspective that bears on the questions of the equity and efficiency of the sequential allocation process. Intrigued

<sup>32</sup>d’Hondt allocations were regarded by nationalists and “Yes Unionists” as a mechanism for achieving inclusive government, but “No Unionists” criticized the system because it was against the normal majoritarian Westminster model—and because it gave Sinn Féin access to office.

by the sequential allocation algorithm described in a conference version of this article, and extending insights from earlier work on (sequential) allocation processes in various contexts (Brams and Davis 1978; Brams, Edelman, and Fishburn 2001; Brams and Straffin 1979; Brams and Taylor 1996, 1999),<sup>33</sup> Brams and Kaplan (2004) have shown that there are substantial potential problems with d'Hondt sequential allocation rules from an *ex post* standpoint. Their work offers an insightful complement to this article. In particular, it provides a powerful theoretical argument against prohibiting post-allocation vote-trading and suggests that, when strategic concerns are taken into account, going early may not be an advantage, especially when preferences are nonseparable (as we have suggested they are likely to be, at least in the Northern Ireland case). Because the focus of our own article is primarily descriptive and because vote trading was explicitly prohibited in the two cases we consider here, we will not further pursue the important theoretical issues raised by the work of Brams and his colleagues.<sup>34</sup>

## Conclusions

In allocating ministerial portfolios among a set of parties whose political strength can be precisely specified by their parliamentary seat shares, divisor methods as sequencing mechanisms provide a solution to the complex bargaining problem of allocating “lumpy” goods, the utility of which may be judged differently by different political actors. In general, we believe that the use of divisor rules for the sequencing of portfolio choices is an intriguing institu-

tional “innovation”<sup>35</sup> to speed the process of coalitional bargaining in a way that generates results that “appear” to be fair.<sup>36</sup> But we have also seen that there are a number of variations that can be offered in such rules, of which two of the most important are the choice of divisor rule and the decision of whether or not to allow *apparentement*.

In studying the 1999 use of sequential allocation processes in Northern and their long-time use in municipal elections in Aarhus, Denmark, from a primarily descriptive standpoint, we have been able to shed light on a number of issues, including the extent to which the assignment process was regarded as equitable by the various parties in both sites; evidence on the strategic incentives induced by the sequential allocation algorithm that tended to create larger than minimal coalitions in Danish city governments; and, for Northern Ireland, evidence on whether or not parties appear to have preference orderings over ministries that are *separable*. In sum, we see our discussion of divisor-rule-based sequential allocation methods as a contribution to several different literatures, including that on bargaining (Young 1991), that on fair division (Balinski and Young 1982; Brams and Taylor 1996, 1999; Pothoff and Brams 1998), that on coalition formation (Laver 1998; Laver and Shepsle 1996), and, not least, that on ethnic conflict resolution.

But it would be a mistake to view algorithmic methods of the sort we have described as only applicable in situations of extreme ethnic or ideological polarization, however appropriate they may be in such situations. In particular, in any situation where there is not a governing coalition that will be given *all* the ministerial portfolios or executive positions, then the bargaining situation may lead to deadlock because the threat is no longer available that, in the absence of agreement to be part of the winning coalition and to agree on a division of the spoils among its members, parties will be excluded from payoffs. In such

<sup>33</sup>We may compare and contrast the divisor rule process for portfolio allocation with rules in professional sports used to determine the order of “draft” picks. Both reflect notions of equity. In professional sports, the determination of draft pick order is specified by some combination of previous team performance and lottery features, and it is designed (in principle) to attempt to equalize the distribution of talent among teams in the league. Divisor-rule sequencing is, of course, based in a straightforward fashion on the (relative) size of each party’s parliamentary seat share, and thus reflects a principle of proportionality. There are also obvious (albeit inverted) parallels between divisor-rule sequencing and (optimal) use of preemptory challenges in jury selection. In the United States and some other countries, defense and prosecution in criminal trials (or plaintiff and defendant in civil trials) have each a fixed number of challenges that allow them to eliminate members from the pool of potential jurors until the pool is reduced in size to the actual number needed for jury service (see Brams and Davis 1978).

<sup>34</sup>We would, however, note that parties are unlikely to have complete information about one another’s *ex ante* preferences, a fact which affects the direct empirical applicability of Brams and Kaplan’s work on sequential allocation rules.

<sup>35</sup>Calling PR-based sequential portfolio allocation an “innovation” despite its use for the better part of the past century in Denmark, as well as its recent use in Northern Ireland and in the European Union seems appropriate. Despite these uses, as best we can judge from personal conversations, this idea is simply not well known to specialists in either the electoral systems literature or the cabinet coalition literature.

<sup>36</sup>See, however, the comments about the *ex post* inefficiencies and inequities of such procedures raised by the work of Brams and colleagues cited to earlier. Moreover, for the case of Northern Ireland, Michael Laver (personal communication, May 2001) and Paul Mitchell (personal communication, April 2001) have each raised the question of whether portfolio allocation outcomes that are not the product of genuine bargaining over a set of allocations as a whole will, in the longer run, be viewed with commitment by the parties involved. In this context, we should note that there is no constructive vote of confidence in Northern Ireland governments created under the procedures of the 1998 Agreement.

situations, found in both our cases as well as in committee assignments within the EU, without some formal mechanism to decide on who gets what, it may be otherwise impossible to proceed.

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